OCR RECOGNISING ACHIEVEMENT

Thursday 21 June 2012 – Afternoon

A2 GCE MATHEMATICS

4724 Core Mathematics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4724
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Simplify

(i) $\dfrac{1-x}{x^2-3x+2}$, **[2]**

(ii) $\dfrac{(x+1)}{(x-1)(x-3)} - \dfrac{(x-5)}{(x-3)(x-4)}$. **[4]**

2 Use integration by parts to find $\int \ln(x+2)\,dx$. **[5]**

3 **(i)** Expand $\dfrac{1+x^2}{\sqrt{1+4x}}$ in ascending powers of x, up to and including the term in x^3. **[6]**

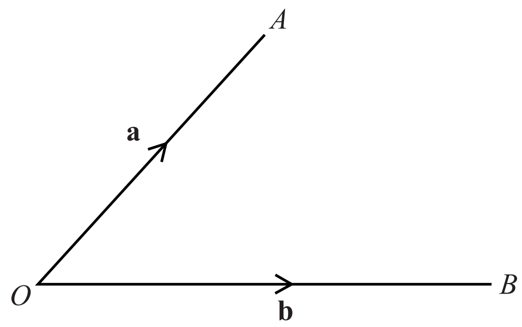
(ii) State the set of values of x for which this expansion is valid. **[1]**

4 Solve the differential equation

$$e^{2y}\frac{dy}{dx} + \tan x = 0,$$

given that $x = 0$ when $y = 0$. Give your answer in the form $y = f(x)$. **[6]**

5



In the diagram the points A and B have position vectors **a** and **b** with respect to the origin O. Given that $|\mathbf{a}| = 3$, $|\mathbf{b}| = 4$ and $\mathbf{a}.\mathbf{b} = 6$, find

(i) the angle AOB, **[2]**

(ii) $|\mathbf{a} - \mathbf{b}|$. **[3]**

6 Use the substitution $u = 1 + \sqrt{x}$ to show that

$$\int_4^9 \frac{1}{1+\sqrt{x}}\,dx = 2 + 2\ln\tfrac{3}{4}.$$ **[7]**

7 Find the exact value of $\int_0^{\frac{1}{6}\pi} (1 - \sin 3x)^2 \, dx$. **[7]**

8 **(a)** Find the gradient of the curve $x^2 + xy + y^2 = 3$ at the point $(-1, -1)$. **[4]**

(b) A curve C has parametric equations

$$x = 2t^2 - 1, \quad y = t^3 + t.$$

(i) Find the coordinates of the point on C at which the tangent is parallel to the y-axis. **[3]**

(ii) Find the values of t for which x and y have the same rate of change with respect to t. **[3]**

9 **(i)** Express $\dfrac{x^2 - x - 11}{(x+1)(x-2)^2}$ in partial fractions. **[5]**

(ii) Find the exact value of $\int_3^4 \dfrac{x^2 - x - 11}{(x+1)(x-2)^2} \, dx$, giving your answer in the form $a + \ln b$, where a and b are rational numbers. **[4]**

10 Lines l_1 and l_2 have vector equations

$$\mathbf{r} = -\mathbf{i} + 2\mathbf{j} + 7\mathbf{k} + t(2\mathbf{i} + 2\mathbf{j} + \mathbf{k}) \quad \text{and} \quad \mathbf{r} = 2\mathbf{i} + 9\mathbf{j} - 4\mathbf{k} + s(\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$$

respectively. The point A has coordinates $(-3, 0, 6)$ relative to the origin O.

(i) Show that A lies on l_1 and that OA is perpendicular to l_1. **[3]**

(ii) Show that the line through O and A intersects l_2. **[4]**

(iii) Given that the point of intersection in part **(ii)** is B, find the ratio $|\overrightarrow{OA}| : |\overrightarrow{BA}|$. **[3]**

Question			Answer	Marks	Guidance
1	(i)		$x^2-3x+2=$ $(x-1)(x-2)$ or $(1-x)(2-x)$ oe	B1	
			Obtain $-\frac{1}{x-2}$ or $\frac{1}{2-x}$ or $\frac{-1}{x-2}$ or $\frac{1}{-(x-2)}$ ISW	B1	Not $\frac{-1}{-(2-x)}$ Accept WW
			If Partial Fractions are used, apply normal mark scheme.	**[2]**	
1	(ii)		Attempt single fraction or 2 fractions with same relevant denom	M1	e.g. $(x-1)(x-4)\left[(x-3)\text{or}(x-3)^2\right]$
			Fully correct fraction(s) before any simplification	A1	
			Relevant numerator = $3x-9$ or $3x^2-18x+27$	B1	Can award if no denominator
			Final answer = $\frac{3}{(x-1)(x-4)}$ or $\frac{3}{x^2-5x+4}$ ISW	A1	
				[4]	
			S.R. If partial fractions are used on each fraction	(M1)	
			$-\frac{1}{x-1}+\frac{2}{x-3}$	(A1)	
			$\frac{2}{x-3}-\frac{1}{x-4}$	(A1)	
			$-\frac{1}{x-1}+\frac{1}{x-4}$ ISW	(A1)	
2			Write (or imply as) $\int 1.\ln(x+2)(\mathrm{d}x)$ (ln x+ ln 2 → M0)	M1	OR: t = ln(x+2) and attempt to connect dx and dt
			Correct 'by parts' 1st stage $x\ln(x+2)-\int\frac{x}{x+2}(\mathrm{d}x)$	A1	$\int te^t\,(dt)$
			Any suitable <u>starting idea</u> for integrating $\frac{x}{x+2}$	M1	Attempt by parts with u = t, $\frac{\mathrm{d}v}{\mathrm{d}t}=e^t$
			[e.g. change num to $x+2-2$ or use substitution $x+2=u$]		
			$\int\frac{x}{x+2}(\mathrm{d}x)==$ $x-2\ln(x+2)$ or $x+2-2\ln(x+2)$	A1	te^t-e^t
			Overall result = $x\ln(x+2)-x+2\ln(x+2)$ [(+c) or (-2+c)] ISW	A1	
				[5]	
			SR: Correct answer with no working	(B2)	

Question			Answer	Marks	Guidance
3	(i)		**The first 5 marks are awarded for expansions of either** $(1+4x)^{-\frac{1}{2}}$ **or** $(1+4x)^{\frac{1}{2}}$		
			Expansion of $(1+4x)^{-\frac{1}{2}}$; First 2 terms $= 1-2x$	B1	<u>Or</u> $(1+4x)^{\frac{1}{2}} = 1+2x$.......
			3rd term = $\dfrac{-\frac{1}{2}.(-\frac{1}{2}-1)}{2}.16x^2$ [Accept $4x^2$ for $16x^2$]	M1	3rd term = $\dfrac{\frac{1}{2}.-\frac{1}{2}}{2}.16x^2$ [ditto]
			$= +\ 6x^2$	A1	$= -2x^2$
			4th term $= \dfrac{-\frac{1}{2}.(-\frac{1}{2}-1).(-\frac{1}{2}-2)}{2.3}.64x^3$ [Accept $4x^3$ for $64x^3$]	M1	4th tm = $\dfrac{\frac{1}{2}.-\frac{1}{2}.-\frac{3}{2}}{2.3}.64x^3$ [ditto]
			$= -20x^3$	A1	$= +\ 4x^3$
			$1-2x+7x^2-22x^3$; $1+ax+(b+1)x^2+(a+c)x^3$	A1 ft **[6]**	ft only $(1+4x)^{-\frac{1}{2}} = 1+ax+bx^2+cx^3$ provided *a*, *b* and *c* attempted and at least one @ M1 obtained
3	(ii)		$\lvert x\rvert < \frac{1}{4}$; $-\frac{1}{4} < x < \frac{1}{4}$; $\{-\frac{1}{4} < x\ ,\ x < \frac{1}{4}\}$ no equality	B1 **[1]**	But not $\{-\frac{1}{4} < x$ <u>**OR**</u> $x < \frac{1}{4}\}$ If choice mark what appears to be the final answer.
4			+/– $\int e^{2y}\,(dy)$ and +/– $\int \tan x\,(dx)$ seen	M1	may be implied later
			$\int e^{2y}(dy) = \frac{1}{2}e^{2y}$	B1	
			$\int \tan x\,(dx) = \ln\lvert\sec x\rvert$ or $-\ln\lvert\cos x\rvert$	B1	Accept ln sec*x* or –ln cos*x*
			Subst *x* = 0, *y* = 0 into their equation containing f(*x*), g(*y*) and c	M1	S.R. Using def integrals: M1 $\int_0^x \quad = \int_0^y$ followed by A2 or A0
			c = $\frac{1}{2}$ **WWW** (or poss $-\frac{1}{2}$ if c on LHS)	A1	
			$y = \frac{1}{2}\ln(1-2\ln\lvert\sec x\rvert)$ or $\frac{1}{2}\ln(1+2\ln\lvert\cos x\rvert)$ oe WWW	A1 **[6]**	Accept omission of modulus

Question			Answer	Marks	Guidance
5	(i)		Use $\cos\theta = \frac{a.b}{\lvert a\rvert\lvert b\rvert}$	M1	
			Obtain $\left(\cos\theta = \frac{6}{12}\right)$ $\theta = 60$ or $\frac{1}{3}\pi$ or 1.05 or better	A1	Better: 1.0471976 (rot)
				[2]	
5	(ii)		Indicate **a** – **b** is vector joining ends of **a** and **b** or equiv $\lvert\mathbf{a}-\mathbf{b}\rvert = \lvert\mathbf{a}\rvert - \lvert\mathbf{b}\rvert$,or anything similar, → M0	M1	
			Use cosine rule correctly on 3, 4 and included (i) angle	M1	Or any other correct method
			Obtain $\sqrt{13}$ or 3.61 or better (No ft from wrong θ)	A1	3.6055513 (rot)
				[3]	
6			Attempt diff to connect d*u* and d*x* or find $\frac{du}{dx}$ or $\frac{dx}{du}$	M1	no accuracy, not just $du = dx$
			Correct e.g. $\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$ or $dx = (2u-2)du$ AEF	*A1	
			Indefinite integral in terms of *u* = $\int \frac{2u-2}{u}(du)$	A1dep*	
			Provided of form $\int \frac{au+b}{u}(du)$, change to $\int a + \frac{b}{u}(du)$	M1	Or by parts
			Integrate to $au + b\ln\lvert u\rvert$ or $au + b\ln u$	A1 ft	
			Use correct variable for limits after attempt at integral of f(u)	M1	i.e. use new values of *u* (usually) or orig values of *x* (if resubst)
			Show as $8 - 2\ln 4 - 6 + 2\ln 3$ (oe) = $2 + 2\ln\frac{3}{4}$ **AG** WWW	A1	Some 'numerical' working must be shown before giving final ans
				[7]	

Question			Answer	Marks	Guidance
7			Satisfactory start method eg attempt square of $(1-\sin 3x)$	M1	Not e.g. $\frac{(1-\sin 3x)^3}{3}$.
			[N.B. The squaring process might include a term $\sin^2 9x$]		
			The next **2** marks are awarded for integrating $-2\sin 3x$		
			Obtain $\int -2\sin 3x\,dx = \frac{2}{3}\cos 3x$	*A1	
			Obtain $-\frac{2}{3}$ or $(...+0...)-(...+\frac{2}{3}...)$	A1dep*	
			The next **3** marks are awarded for integrating $\sin^2 3x$		or for integrating $\sin^2 ax$ where a = 6 or 9 **only**
			Use $\sin^2 3x = k(+/-1+/-\cos 6x)$	M1	$\sin^2 ax = k(+/-1+/-\cos 2ax)$
			Correct version = $\frac{1}{2}(1-\cos 6x)$	A1	Correct = $\frac{1}{2}(1-\cos 2ax)$
			$\int \cos 6x\,dx = \frac{1}{6}\sin 6x$, seen anywhere, indep	B1	or $\int \cos 2ax\,dx = \frac{1}{2a}\sin 2ax$
			Final answer = $\frac{1}{4}\pi + their -\frac{2}{3}$	A1	Check that the $\frac{1}{4}\pi$ is from $\left[\frac{3}{2}x - \frac{1}{12}\sin 6x\right]_0^{\frac{1}{6}\pi}$
				[7]	

Question			Answer	Marks	Guidance
8	(a)		$\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$	B1	
			$\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$	B1	
			Substitute $(-1,-1)$ for (x, y) & attempt to solve for $\frac{dy}{dx}$	M1	or solve then substitute
			Obtain $\frac{dy}{dx} = -1$ WWW	A1	
				[4]	
8	(b)	(i)	Tangent parallel y-axis $\rightarrow \frac{dx}{dt} = 0$ or $\frac{dy}{dx} \rightarrow \infty$ or $\frac{dy}{dx} = \infty$	M1	Accept clear intention
			Obtain $t = 0$	A1	
			$(-1,0)$ with no other possibilities	A1	Accept $x = -1, y = 0$
				[3]	
8	(b)	(ii)	State or imply or use $\frac{dy}{dt} = \frac{dx}{dt}$	M1	
			Produce $3t^2 + 1 = 4t$ oe	A1	
			$t = \frac{1}{3}$ or 1	A1	
				[3]	

Question			Answer	Marks	Guidance
9	(i)		$\frac{A}{x+1}+\frac{B}{x-2}+\frac{C}{(x-2)^2}$	B1	i.e. correct partial fractions
			$A(x-2)^2+B(x+1)(x-2)+C(x+1)=x^2-x-11$	M1	or equivalent identity or method
			$A=-1$	A1	B1 if cover up method used
			$B=2$	A1	
			$C=-3$	A1	B1 if cover up method used
				[5]	
			Special Cases The problems arise when we see how candidates deal with the denominator $(x-2)^2$: $\frac{A}{x+1}+\frac{Bx+C}{(x-2)^2}$; allow B1 for PF format, M1 for associated identity, B1 for $A=-1$ (max 3) $\frac{A}{x+1}+\frac{B}{x-2}+\frac{Cx+D}{(x-2)^2}$; allow B1 for PF format, M1 for assoc identity, B1 for $A=-1$ (max 3) $\frac{A}{x+1}+\frac{Bx}{(x-2)^2}$; allow B0 for PF format, M1 for associated identity (max 1, even if $A=-1$) $\frac{A}{x+1}+\frac{B}{(x-2)^2}$: allow B0 for PF format, M1 for associated identity (max 1, even if $A=-1$)		
9	(ii)		No marks are to be awarded for integrating a fraction with a zero numerator. Irrespective of the format used for the Partial Fractions in part (i), award marks as follow:		
			$\int\frac{\lambda}{x+1}\mathrm{d}x=\left(\lambda \text{ or } \frac{1}{\lambda}\right)\ln(x+1)$ or….	B1	$\int\frac{\lambda}{x-2}\mathrm{d}x=\left(\lambda \text{ or } \frac{1}{\lambda}\right)\ln(x-2)$
			$\int\frac{\mu}{(x-2)^2}\mathrm{d}x=-\left(\mu \text{ or } \frac{1}{\mu}\right).\frac{1}{x-2}$	B1	
			$-\frac{3}{2}$	B1 ft	ft $\frac{C}{2}$
			$.....+\ln\frac{16}{5}$ ISW for either term	B1 ft	ft $.....+\ln\left\{\left(\frac{5}{4}\right)^A .2^B\right\}$
				[4]	

Question			Answer	Marks	Guidance
10			If MR, mark according to the scheme & follow-through from candidate's data. Award M, A & B marks (where possible) & apply penalty of 1 mark (by withholding one A mark in the question). E.g. in (i), product to be 'correct' & 'not perpendicular' to be stated.		
	(i)		α. Full justification that $t=-1$. May be 'by inspection'. [No equations not satisfied by $t=-1$ to be shown] ['unusual' attempts must be carefully checked; if convinced, award the B1 e.g. displacement vector between (-3**i** + 6**k**) and (-**i** + 2**j** + 7**k**) = ±(2**i** + 2**j** + **k**)]	B1	No other t = to be mentioned
			β. Consider scalar product $\begin{pmatrix}-3\\0\\6\end{pmatrix}.\begin{pmatrix}2\\2\\1\end{pmatrix}$	M1	
			Show **– 6 + (0) + 6** = 0 and somewhere state perpendicularity oe	A1	
			[If $\cos\theta=\frac{\mathbf{a}.\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ quoted, ignore accuracy of work involving $\lvert\mathbf{a}\rvert$ and $\lvert\mathbf{b}\rvert$]	**[3]**	
10	(ii)		Use **r** = v (-3**i** +6**k)** and ℓ_2	*M1	or $(-3\mathbf{i}+6\mathbf{k})$ + v$(-3\mathbf{i}+6\mathbf{k})$
			Attempt to produce at least two relevant equations	M1dep*	
			Solve two equations & produce $(v,s)=\left(\frac{1}{3},-3\right)$ soi	A1	$(v,s)=\left(-\frac{2}{3},-3\right)$
			Demonstrate clearly that these satisfy third equation	B1 **[4]**	Numerical proof required
10	(iii)		Method for finding $\lvert\overrightarrow{OB}\rvert$ or $\lvert\overrightarrow{OA}\rvert$ or $\lvert\overrightarrow{AB}\rvert$	M1	Method for finding $\overrightarrow{OB}$ or $\overrightarrow{BO}$ or $\overrightarrow{AB}$ or $\overrightarrow{BA}$
			$\lvert\overrightarrow{OB}\rvert=\sqrt{5}$ or $\lvert\overrightarrow{OA}\rvert=\sqrt{45}$ oe or $\lvert\overrightarrow{BA}\rvert=\sqrt{20}$ oe	A1	$\overrightarrow{OB}=\begin{pmatrix}-1\\0\\2\end{pmatrix}$ or $\overrightarrow{BA}=\begin{pmatrix}-2\\0\\4\end{pmatrix}$
			Obtain 3:2 oe	A1 **[3]**	Answer 3:2 WW → B3